Scaling Technique in Tempered Stable Processes and Its Application to Financial Data Analysis

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The Normal Tempered Stable (NTS) distribution has been used as one of the most popular models in finance since it can describe the fat-tails and asymmetric features of financial return data. In Figure 1, we can see that the distribution of the daily S&P 500 index returns has fat-tails. Hence, the normal distribution fails to fit the data while the NTS distribution fits well. The NTS is defined by the characteristic function (Ch.f), and the probability density function (pdf) of NTS is not given by a closed-form solution. For that reason, the fast Fourier transform (FFT) method has been used to obtain the pdf and cumulative distribution function (CDF). In this research, we develop a new efficient method, named the scaling technique, to calculate the PDF and CDF of NTS distribution using FFT from the Ch.f. The basic idea of the scaling technique is to calculate the PDF and CDF of general NTS distribution by scaling for the PDF and CDF of the standard NTS distribution. Using the new method, we obtain more accurate PDF and CDF faster than the traditional FFT method. For example, the PDF of NTS distribution has many numerical errors using the traditional FFT method for some special parameter set ($\alpha = 0.5, \theta = 10, \beta = -3, \gamma = 0.1, \mu = 0, \Delta t = 1/250$) as the left plate of Figure 2, but those errors disappeared if we use the scaling technique as the right plate of Figure 2. By applying the scaling technique, we obtained better parameter estimation result for the intraday return of IBM as Figure 3. The scaling technique can be used for the Monte-Carlo simulation method as Figure 4. The simulation method will be applied for the European option pricing. In the poster session, we will show the theoretical details of the scaling technique and provide more statistical evidence on why the method is better. We present many applications of the scaling technique of the NTS distribution in finance, such as risk assessment and option pricing.

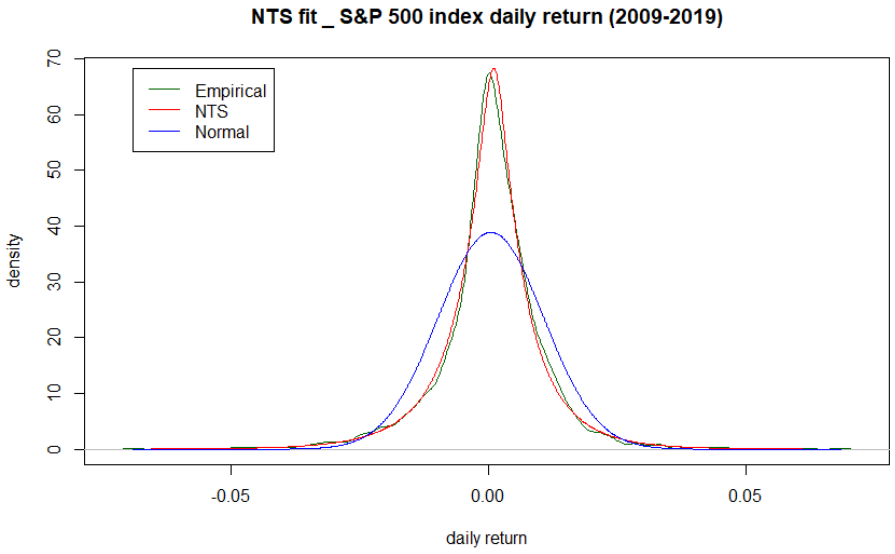

Figure 1. Fattails of empirical distribution and NTS fit

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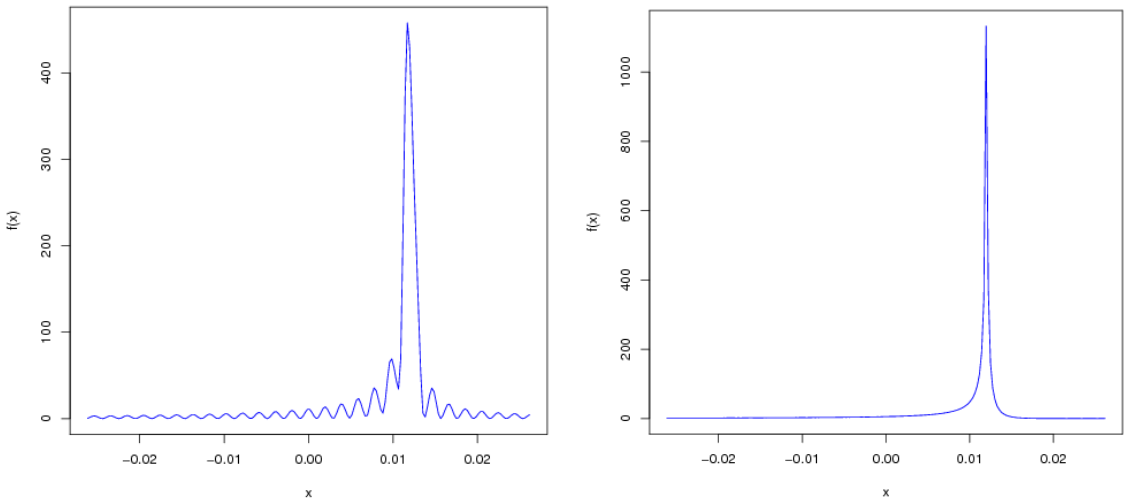

Figure 2. pdfs of the NTS distribution using traditional FFT method (Left) and using the scaling technique (right), respectively.

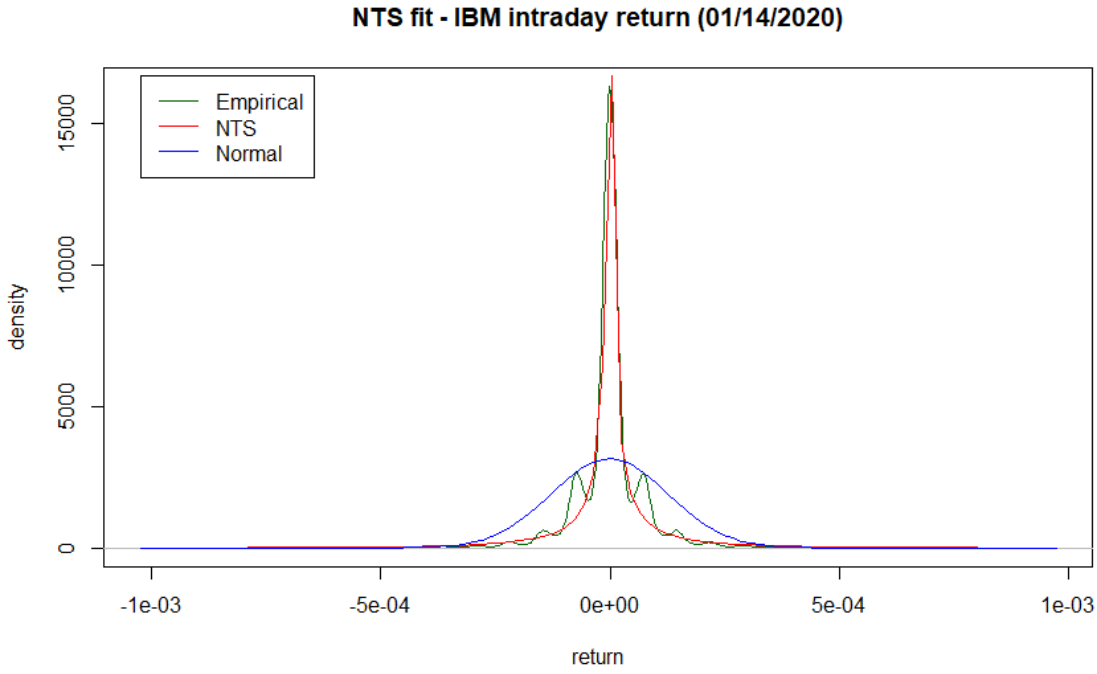

Figure 3. intraday return of IBM stock and NTS fit

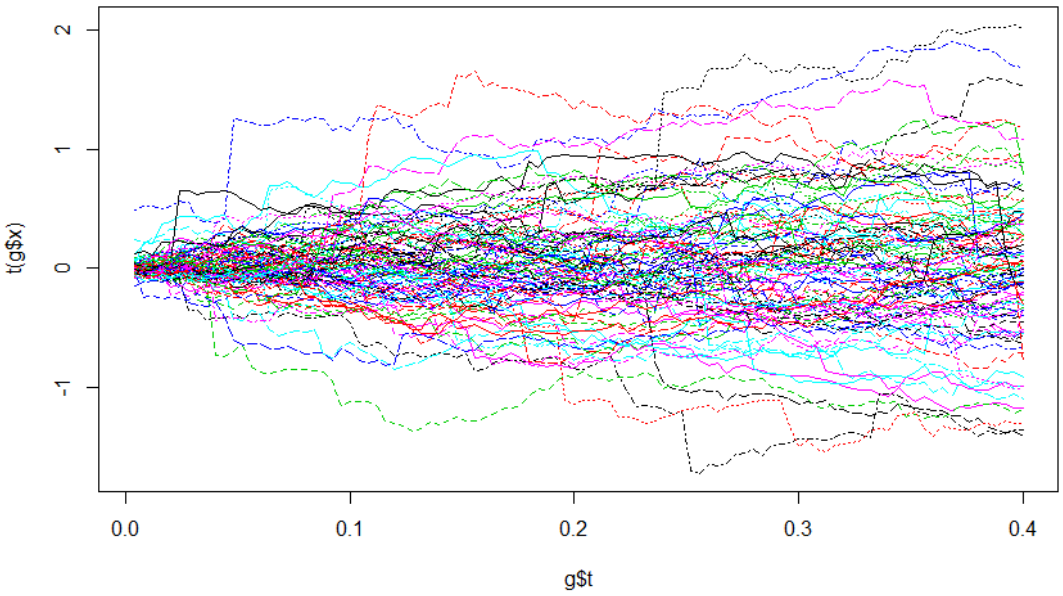

Figure 4. Simulation NTS process